Math 128A: Homework 3

Due: July 5

1. Find a sequence $\{p_n\}_{n\geq 0}$ that converges to 0 cubically and prove that it does so.

2. Consider the sequences

$$\text{(i) } p_n = 2 + \frac{1}{p_{n-1}}, \quad p_0 = 2; \qquad \text{(ii) } p_n = p_{n-1}(2 - \pi p_{n-1}), \quad p_0 = 1/2.$$

   (a) Prove that both (i) and (ii) converge and find their limits.

   (b) For both (i) and (ii), find α, the order of convergence, and $\lim_{n\to\infty} \frac{|p_{n+1}-p|}{|p_n-p|^\alpha}$.

   (c) Given any $\epsilon > 0$, find the number of steps (in terms of ϵ) necessary to guarantee absolute errors of less than ϵ for (i) and (ii).

3. (a) Write a MATLAB function

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function [p,imax] = SecMethod(f,p0,p1,tol,Nmax)
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   that implements the secant method.

   (b) Write a MATLAB function

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function [p,imax] = FalsePos(f,p0,p1,tol,Nmax)
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   that implements the method of false position (i.e., *regula falsi*).

4. Solve the following equations accurate to 10^{-8} using (i) Newton's method, (ii) the secant method and, (iii) the method of false position.

   (a) $x^2 - 4x + 4 - \ln(x) = 0$ in $[1, 2]$.

   (b) $x^x = 50$ in $[3, 4]$.

   (c) $2x + 3\cos(x) - e^x = 0$ in $[-1, 1]$.

5. The equation

$$x^4 - 6x^3 + 12x^2 - 10x + 3 = 0$$

   has a solution for $x = 1$.

   (a) Find the multiplicity of this solution.

   (b) Apply Newton's method to this problem. Is the convergence quadratic?

   (c) Apply the Modified Newton's method to this problem and analyze the convergence.

6. The following sequences converge to 0. Use Aitken's acceleration to generate $\{\hat{p}_n\}$. Give the smallest N such that $|\hat{p}_N| < 10^{-8}$.

   (a) $p_n = \ln(2) + \sum_{i=1}^{n} \frac{(-1)^i}{i}$.

   (b) $p_n = \frac{\pi^2}{12} + \sum_{i=1}^{n} \frac{(-1)^i}{i^2}$.

7. Use Steffensen's method with $p_0 = 3$ to compute an approximation to $30^{1/3}$ accurate to within 10^{-8}.

8. Use Horner's method to run the first two steps of Newton's method on

$$2x^3 - 5x^2 + 3x - 2 = 0$$

   starting from $p_0 = 1$.

9. Construct the quadratic polynomials $p(x)$ that interpolate the following functions f at $x_0 = -1$, $x_1 = 0$ and $x_2 = 1$ and find upper bounds for $|f(x) - p(x)|$ for $|x| \leq 1$.

   (a) $f(x) = \sin(\pi x/2)$.

   (b) $f(x) = \frac{1}{1+x^2}$.

Submission Details: Turn in all problems on paper except problem 3. Upload your MATLAB files for that to bCourses.